An Experimental Investigation of Natural Convection from an Isothermal Horizontal Plate

A.M. Clausing
Professor, Mem. ASME

J.J. Berton
Department of Mechanical and Industrial Engineering,
University of Illinois at Urbana-Champaign, Urbana, IL 61801

An investigation of natural convection from a heated, upward-facing, square, horizontal plate to a surrounding gas medium is described in this paper. The results of the experimental investigation provide an improved correlation for the natural convection regime by accounting for variable property effects and extend the applicable Rayleigh number (Ra) range of the correlation over previous research. The large Rayleigh number regime is emphasized. The value of the Richardson number (Ri) at which combined convection influences become important is also determined. The ratio of the plate wall temperature T_w to the ambient temperature T_∞ is incorporated into the Nusselt number correlation in order to account for variable property influences. A cryogenic heat transfer tunnel, with test section temperatures that are varied between 80 K and 310 K, is used to help deduce the influences of the relevant parameters. The ranges of the dimensionless parameters investigated are $2\times 10^8 < \mathrm{Ra} < 2\times 10^{11}$ and $1 < T_w/T_\infty < 3.1$.

Introduction

When the ratio of the absolute temperature of a heated object to the absolute ambient temperature, T_w/T_∞ , is appreciably greater than unity, the thermophysical properties of the fluid within the thermal boundary layer vary markedly. Most analytical and experimental research performed during the past few decades has ignored all property variations except the basic density differences that generate the buoyancy force. This approach, which will be referred to as the constant-property case, is valid as long as T_w/T_∞ is near unity. Many applications exist, however, where T_w/T_∞ is much greater than unity. In these applications, the reference temperature that is chosen to evaluate the properties in constant-property correlations can strongly influence the values of the dimensionless parameters and the predicted rate of heat transfer.

The results of previous experimental work using the constant-property assumption differ greatly. While all researchers arrive at the same Rayleigh number exponent, their correlations differ by as much as 25 percent (see, for example, Bosworth, 1952; Hassan and Mohammed, 1970). Analytical solutions to the horizontal plate problem (see, for example, Rotem and Claasen, 1969; Pera and Gebhart, 1973) do not account for the effects of turbulence within the boundary layer and consequently are only applicable to low Rayleigh number, laminar flow.

Published in the *ASME Journal of Heat Transfer*, November, 1989.

The results reported in this study appreciably extend the horizontal plate correlation. Specifically, data are given for Rayleigh numbers more than six times greater than those reported by Fishenden and Saunders (1950), and up to 2000 times greater than those given by other researchers (see: Hassan and Mohammed, 1970; Fujii and Imura, 1972; Lloyd and Moran, 1974; Al-Arabi and El-Riedy, 1976). Also, all of the data in the literature are for T_w/T_∞ near unity, whereas in this study the range $1 < T_w/T_\infty < 3.1$ is investigated. The accuracy of the resulting correlation is greatly enhanced by deducing the influence of this additional parameter.

A matter of concern for researchers performing natural convection experiments is the influence of extraneous drafts or currents on their measurements. Experimenters go to great lengths to ensure that drafts do not adversely effect their results. In this study, the Richardson number, Ri, at which inertial influences become significant is resolved.

One reason for the absence of data in these important regimes is the difficulty in generating large Rayleigh numbers. Another problem is obtaining large values of T_w/T_∞ without masking the results by radiative heat transfer. For these reasons, a variable ambient temperature cryogenic facility, which was constructed at the University of Illinois at Urbana-Champaign (UIUC), is used in the investigation.

Experimental Apparatus and Procedure

The UIUC facility is a variable ambient temperature tunnel that can operate with test section temperatures between 80 K and 310 K. By reducing the temperature of the working fluid, dramatic changes occur in the thermophysical properties of the gas and large increases in the Rayleigh number can be realized. In order to achieve large Rayleigh numbers, previous investigators had to heat their models to temperatures far above 300 K. For example, Fishenden and Saunders (1950) used surface temperatures above 800 K. This necessitates the separation of the convective component of heat transfer from a large radiative component. In contrast, the radiative heat transfer in a high Rayleigh number test at cryogenic temperatures is typically less than one percent of the total heat transfer. The variable T_∞ feature of the UIUC facility also enables one to cover large ranges of the relevant dimensionless groups such as T_w/T_∞ , Re, Ra, and Ri without changing models. Other advantages of the use of a cryogenic environment are described by Clausing (1982).

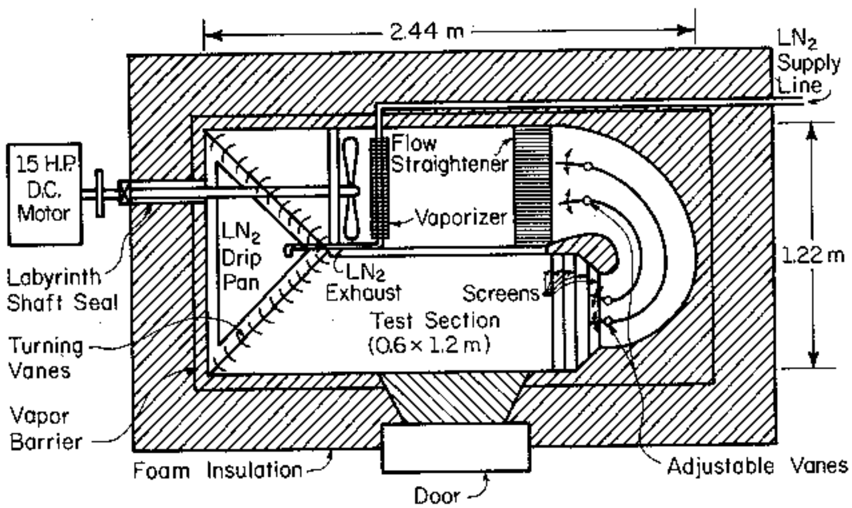

Fig. 1: Cross-sectional top view of cryogenic facility

The UIUC cryogenic facility, which was extensively modified during the past three years in order to improve the quality of its flow field and it cooling characteristics, is illustrated in Fig. 1. The tunnel has a rectangular test section with a height of 1.2 m and a width of 0.6 m. It is a variable-speed, recirculating tunnel with a maximum test section velocity of 6 m/s. The test section temperature T_∞ is reduced for the cryogenic tests by the vaporization of liquid nitrogen in a newly installed, cascading, finned tube heat exchanger located just downstream of two 0.5-m-dia cast aluminum fans. The tunnel is carefully sealed and insulated with 0.4 m of urethane insulation. This insulation system, in combination with the thermal mass that lies inside the insulating envelope, results in a negligible variation of T_∞ during a test. Gaseous nitrogen is used as the working fluid in cryogenic tests, and air is used in experiments that are conducted with $T_\infty > 290$ K. A combination of four tuned vanes and a series of five screens (see Fig. 1) is used to obtain a flow field throughout the test section with a velocity uniformity of 1.5 percent and a turbulence intensity of 1.7 percent.

In order to generate large Rayleigh numbers, the square plate is represented by an L by $L/2$ plate where $L = 0.6$ m (see Fig. 2). That is, the vertical line of symmetry that bisects the plate is replaced by a vertical adiabatic wall. The comparison of natural convection tests conducted with and without an additional vertical wall that bisects the $L/2 \times L$ plate shows that the extra viscous shear induced by a wall of symmetry has a negligible influence on the heat transfer from the plate. The $L/2 \times L$ model is divided into four thermally isolated calorimeters of equal mass and area in order to resolve areas of different convective heat transfer rates (see Fig. 2). Each calorimeter is 0.3 m × 0.15 m × 7.9 mm and weighs 0.98 kg. The calorimeters are highly-polished, 6061-T6 aluminum plates. They are individually heated by thin foil resistance heaters of negligible thermal mass, which are mounted on the back sides. Directly underneath the model and the heaters is a 25-mm board of urethane insulation, a heated copper guard, and a second 25-mm board of insulation. The guard is maintained at the same temperature as the calorimeters to eliminate conductive heat transfer in that direction. The model is instrumented with eighteen 30-gage copper-constantan, special-accuracy thermocouples. Automatic ice point references were used to hold the thermocouple reference junctions at 0°C (± 0.02°C). The temperature distribution in the test section of the tunnel is determined with a grid of six identical thermocouples.

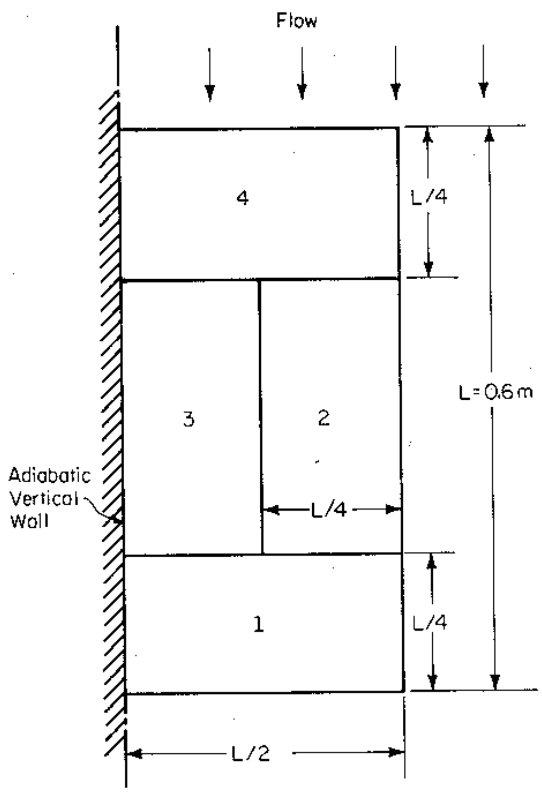

Fig. 2: Sketch of model illustrating calorimeter arrangement

In a typical test, the four calorimeters are heated to an isothermal initial condition. The heaters are then turned off, and the flow, if desired, is initiated. The data acquisition system scans the 24 channels continuously at a rate of 12 channels per second, records, the converted thermocouple readings in the memory of a microcomputer, averages the temperature readings of each of the four plates as well as other ensembles of interest, and graphically displays the specified temperatures and/or average temperatures. At the end of a test, the data are written on a diskette and subsequently are uploaded to a mainframe computer for further processing and plotting.

The rate of convective heat transfer from each plate is determined at a time after quasi-steady conditions are reached but before thermal gradients within the plate become significant. The rate of heat transfer is determined from the rate of change of the internal energy of the respective calorimeter; that is, the convective heat transfer coefficient is determined from the energy balance

$$h = \frac{\dfrac{-mc_p}{A}\dfrac{dT_w}{dt} - \sigma\varepsilon\left(T_w^4 - T_\infty^4\right)}{T_w - T_\infty}$$

where m is the mass of the calorimeter, c_p its specific heat, A is the heat transfer area, ε is the emissivity for polished aluminum, and σ is the Stefan-Boltzmann constant. The heat transfer surface is assumed to be gray, and the surroundings are assumed to be isothermal and black. The variations of both c_p and ε with calorimeter temperature are taken into account. The time derivative of the calorimeter temperature is determined with a central difference quotient. The average heat transfer coefficient over the entire plate is determined by averaging the individual coefficients of the four identical plates.

Figures 3 and 4 show typical results for a 150 s, natural convection test with a Rayleigh number of 5.94×10^{10}, an ambient temperature of 98.2 K, and a temperature ratio T_w/T_∞, of 2.00. The smoothness of the temperature versus time curves attests to the accuracy of the experiment.

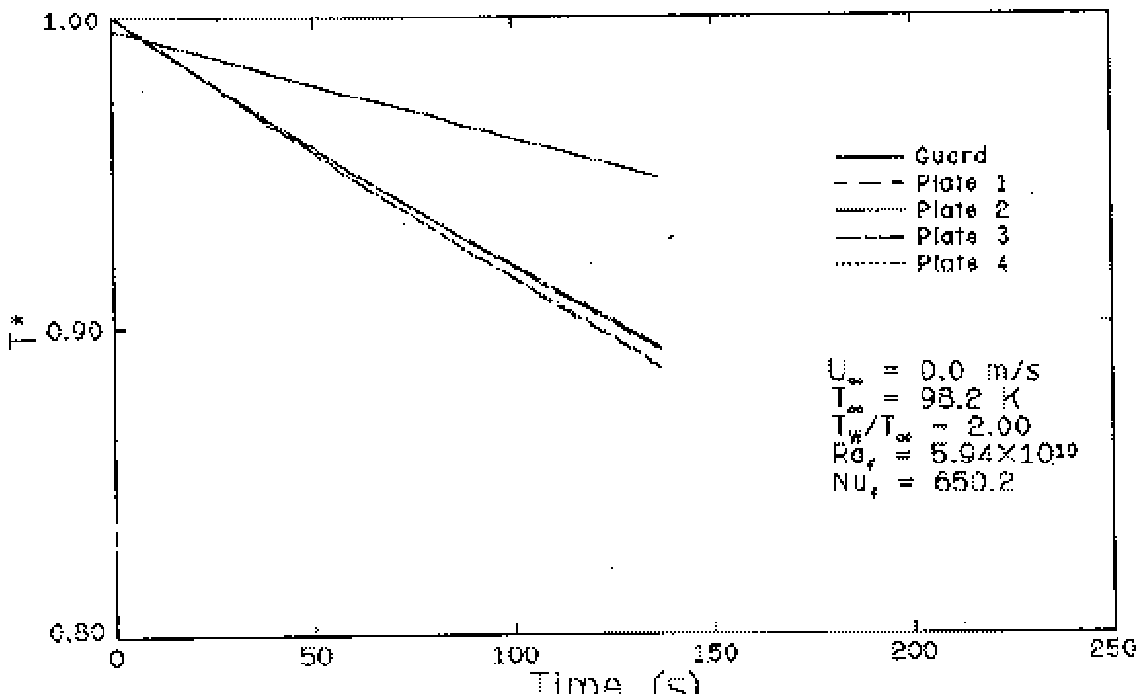

Fig. 3: Dimensionless calorimeter temperatures for a typical test

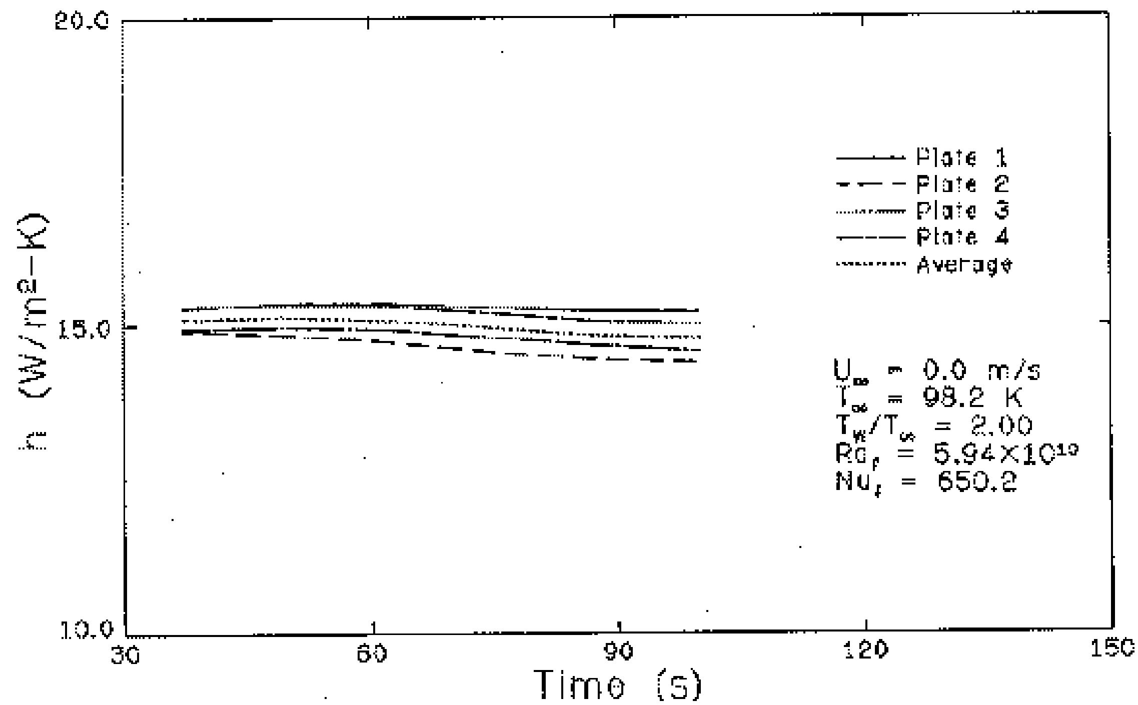

Fig. 4: Individual heat transfer coefficients for a typical test

A Fluke Model 2240B automatic datalogger, in conjunction with a Texas Instruments Portable Professional Computer, is used for data collection. The datalogger measures the thermocouple potentials to within 0.1 μV. After considering the disturbing influences of the imperfect ice reference units, the voltage-to-temperature conversion error, and the datalogger accuracy, the temperature measurements are assigned a relative uncertainty of 0.05 K. Following an examination of the thermophysical property data of air, nitrogen, and aluminum, 2 percent uncertainties are assigned to the specific heat, density, viscosity, and conductivity data. An 8 percent uncertainty is assigned to the emissivity data. Uncertainties in the measurements of the characteristic lengths, heat transfer area, model mass, time, and the ambient temperature (due to the small but inevitable temperature stratification at cryogenic temperatures) are judged to be 0.001 m, 0.01 m^2, 0.001 kg, 0.5 s, and 2 K, respectively. The results of an uncertainty analysis show that the uncertainties in the heat transfer coefficients and Rayleigh numbers are 2 percent and the uncertainty in the Nusselt number is 3 percent. Further details of the experimental procedure can be found in Berton (1986).

Similitude Considerations

The set of dimensionless groups that influence natural and combined convection is deduced by an examination of the governing equations. The simplifying assumptions are *(i)* a steady flow of a Newtonian fluid, *(ii)* a perfect gas, *(iii)* negligible viscous dissipation and work done by compression, *(iv)* the dependent variables c_p/c_{pr}, υ/υ_r, and k/k_r are general functions of the dimensionless temperature ratio T/T_r, *(v)* isothermal gaseous surroundings at T_∞ , and *(vi)* and isothermal surface at T_w. The Boussinesq approximation is not used. The dimensional analysis with these assumptions shows that the average Nusselt number for natural convection is dependent on

$$\text{Nu} = f_1 (\text{Ra}, \text{Pr}, T_w/T_\infty) \qquad (2)$$

For combined convection, one obtains

$$\text{Nu} = f_2 (\text{Ra}, \text{Pr}, \text{Ri}, T_w/T_\infty) \qquad (3)$$

The determination of the function f_2 is not addressed in this paper; however, the limiting Richardson number at which combined convection influences become negligible is determined. The Prandtl number influence is also not resolved; thus, the results are only applicable to gases with a Pr of approximately 0.7. Experience has shown that for values of T_w/T_∞ near unity, the form of equations (2) and (3) is valid for both laminar and turbulent natural convection.

The influence of variable properties is taken into account in equations (2) and (3) with the parameter T_w/T_∞ . This contrasts with commonly used procedures -- the reference property method and the property ratio method. Following the procedure used by Clausing (1983), the form of the correlation in the natural convection limit is assumed to be

$$\text{Nu}_r = g(\text{Ra}_r) \bullet f_r (\text{Ra}_r, T_w/T_\infty) \qquad (4)$$

where $g(\text{Ra}_r)$ is defined as the constant-property correlation, that is, $f_r (\text{Ra}_r, 1) = 1$. Although equation (4) is similar to the property ratio method, the stringent constraint of having to account for variable property influences with a function of a single property ratio is removed. The function f is, of course, a function of the reference temperature T_r, which is used in the evaluation of Nu and Ra; hence, the subscript r is employed. In this paper, the function f is evaluated for three reference temperatures: T_w, $T_f \equiv (T_w + T_\infty)/2$, and T_∞ .

Results

An examination of the natural convection data from this investigation showed that

$$\text{Nu} \sim \text{Ra}^{1/3},\ 2 \times 10^8 < \text{Ra} < 2 \times 10^{11} \qquad (5)$$

over the specified Ra range of the data, if T_w/T_∞ were near unity. The data in this Rayleigh number range also showed that f is a strong function of only T_w/T_∞ . Hence, it was assumed that the variable property influence could be accurately represented by a second-degree polynomial in T_w/T_∞ . A least-squares, second-degree fit of the experimental data with the constraints $f(1) = 1$ and $\text{Nu} \sim \text{Ra}^{1/3}$ gives

$$\mathrm{Nu}_r = 0.140\ \mathrm{Ra}_r^{1/3}\ [\ a_1 + a_2(T_w/T_\infty) + a_3(T_w/T_\infty)^2\ ] \quad (6)$$

where the constants a_i are given in Table 1 for the three different reference temperatures: T_w, T_f, and T_∞ The three variable property correlations and the corresponding experimental data are graphically illustrated in Fig. 5. The coefficient of 0.140 in the constant-property correlation, g(Ra), agrees with the coefficient reported by Fishenden and Saunders (1950), who used the same Rayleigh number exponent. They used elevated pressures in order to obtain large Rayleigh numbers.

Reference Temperature	a_1	a_2	a_3
Wall	0.433	0.626	-0.0581
Film	0.823	0.179	-0.0011
Ambient	1.212	-0.254	0.0405

Table 1: Constants in variable property correlation, f_r (see equation (6))

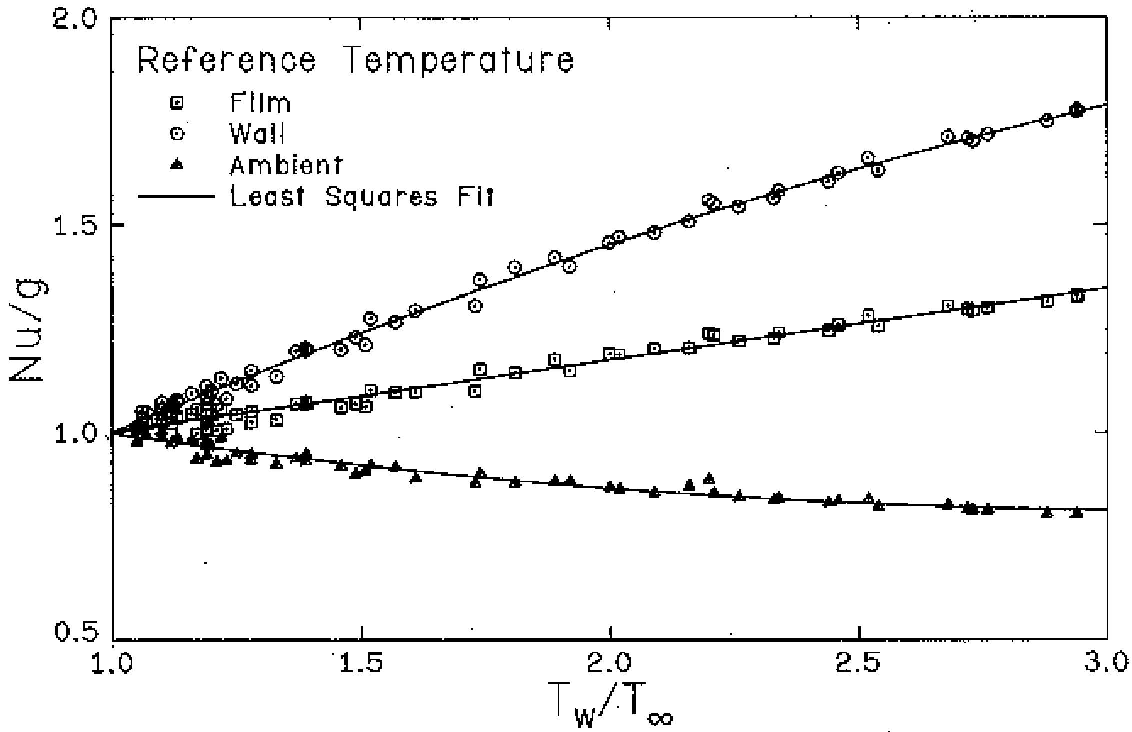

Fig. 5: Variable property correlation, natural convection data

The choice of reference temperature has, by definition, no influence on the constant-property correlation g(Ra). The choice of reference temperature does, however, have a significant influence on $f(T_w/T_\infty)$, which can be seen in Fig. 5. The function f is greater than unity if the wall or film temperature is used as the reference temperature, whereas it is less than unity if the properties are based on the ambient temperature. This result contrasts with the correlations from a similar experiment, natural convection from an isothermal vertical surface, which were reported by Clausing (1983). For the case of a vertical plate, the function f is greater than unity regardless of the reference temperature used. The variable property influence is also significantly greater in the case of the vertical plate. In the present study, the function f is remarkably linear, which is evident from the relatively small values of the coefficient a_3. This is again in contrast with the marked curvature of f for the case of the vertical plate (Clausing, 1983). The linear nature of the function f suggests the use of the reference temperature method of compensating for variable properties. In this method, a reference temperature at which all properties are evaluated is chosen such that the function g(Ra) achieves the maximum degree of correlation. If the reference temperature is chosen to be

$$T_r = T_w - 0.83(\, T_w - T_\infty), 1 < T_w/T_\infty < 3 \qquad (7)$$

then the value of $f(T_w/T_\infty)$ is found to be unity and no property corrections are necessary for the range of temperature ratios investigated. Although the reference temperature method is widely used, it should be stressed that for other geometries, particularly in the case of vertical plates, the technique is not successful.

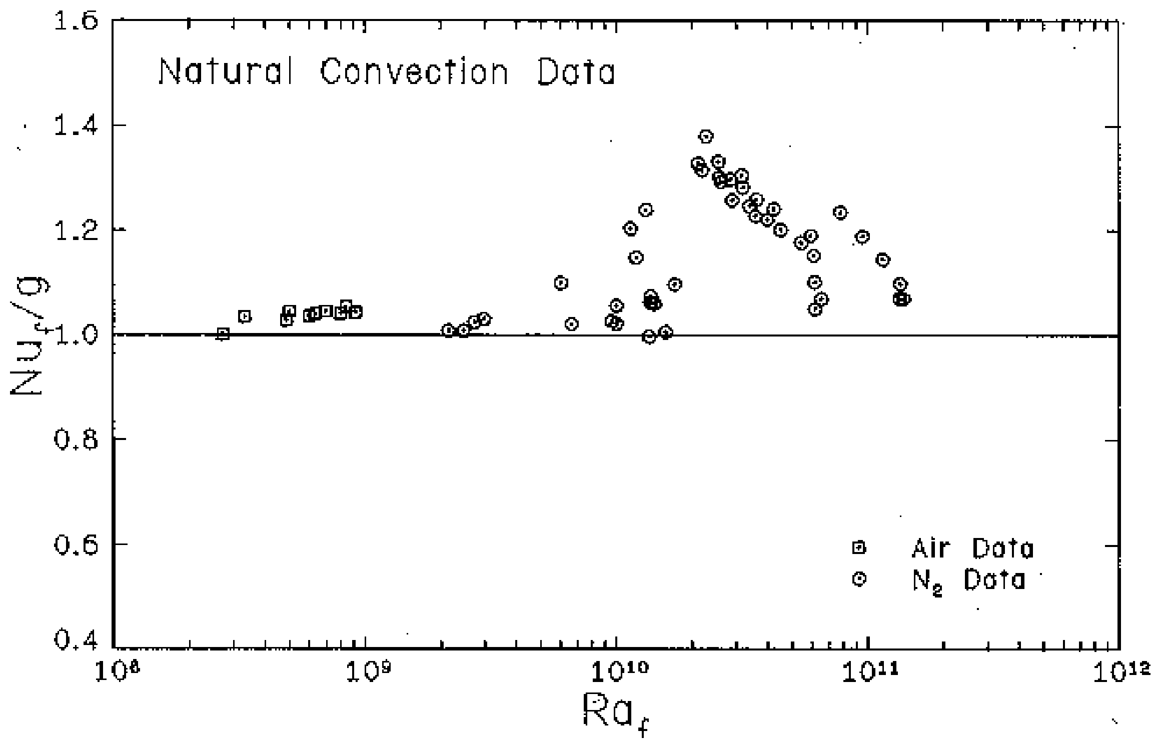

Fig. 6: Degree of correlation, including influence of only Ra

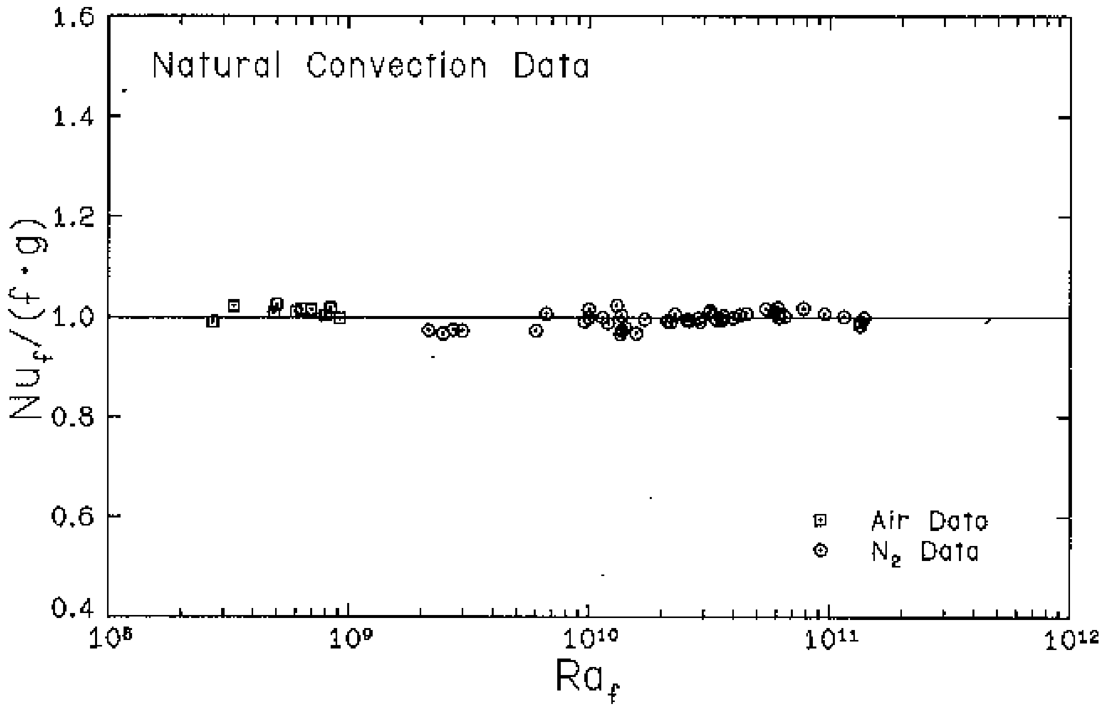

Fig. 7: Comparison between natural convection data and proposed correlation

The agreement between Correlation (6) and the experimental data is illustrated in Figs. 6 and 7. Figure 6 shows the degree of correlation obtained with the constant-property correlation. This figure is a plot of the Nu_f divided by the function $g(Ra_f)$ versus Ra_f. The inability of the constant-property correlation to predict the experimental results is evident. Deviations as large as 40 percent are present. Figure 7, a graph of $Nu_f/(g \bullet f_f)$ versus Ra_f, clearly shows excellent agreement between the 56 data points and Correlation (6). The maximum deviation of any data point is only 3.3 percent.

The combined convection regime near the natural convection limit was also investigated in order to determine the Richardson number at which combined convection influences become significant. The data showed that in order for the free-stream velocity to increase the convective heat transfer by more than one percent, the Richardson number Ri_f must be less than 87. The increase is less than 5 percent if $Ri_f > 33$. The influences of the 1.7 percent free-stream turbulence intensity and the distortion in the flow field, due to the finite model thickness and the presence of the vertical adiabatic wall, were estimated to be too large to delineate the functional relationship between the Nusselt number and the Richardson number accurately. Hence, these data are not presented in this paper (see Berton, 1986, for further information).

Conclusions

The following conclusions are drawn from the results of this investigation:

1. The variable property influence can be accounted for with a function of only T_w/T_∞ over the Rayleigh number range $2 \times 10^8 < \text{Ra} < 2 \times 10^{11}$. The results for natural convection from a vertical plate (Clausing, 1983), gave: $f_f(\text{Ra}, T_w/T_\infty) = 1$ in the laminar region, f_f equal to a function of both Ra and T_w/T_∞ in the transitional regime, and f_f equal to a function of only T_w/T_∞ in the turbulent regime. f_f reached values as high as 2 at $T_w/T_\infty = 2$ in the turbulent regime. The dependency of f_f on only T_w/T_∞ in this investigation indicates that all data lie in the turbulent domain. This is substantiated by the turbulent data reported by Fishenden and Saunders (1950), Fujii and Imura (1972), Lloyd and Moran (1974), and AI-Arabi and EI-Riedy (1976).

2. The Nusselt number in the turbulent regime is strongly affected by property variations. Correlations proposed in the literature, such as the Fishenden and Saunders correlation, equation (3), are acceptable only if T_w/T_∞ is near unity. For example, the Nusselt number at $T_w/T_\infty = 3.0$ is 35 percent greater than the value predicted by the Fishenden and Saunders constant-property correlation.

3. Due to the linear nature of $f(T_w/T_\infty)$, a reference temperature can be selected that forces this function to be unity for the entire range of temperature ratios investigated. This reference temperature is $T_r = T_w - 0.83(T_w - T_\infty)$.

4. The combined convection data indicate that small to moderate drafts, such as doors opening or people moving about, can have an effect on the rate of heat transfer in "free" convection. Further evidence of combined convection influences is the great effort taken by some investigators of natural convection phenomena to eliminate such influences in their experiments (see discussions given by Clausing, 1983).

Acknowledgments

The authors gratefully acknowledge the financial support of the National Science Foundation in sponsoring this research.

References

1. AI-Arabi, M., and EI-Riedy, M. K., 1976, "Natural Convection Heat Transfer From Isothermal Horizontal Plates of Different Shapes," *International Journal of Heat and Mass Transfer*, Vol.19, pp.1399-1404.
2. Berton, J.J., 1986, "An Experimental Investigation of Natural and Combined Convection From an Isothermal Horizontal Plate," M.S. Thesis, University of Illinois, Urbana, IL.
3. Bosworth, R. L. C., 1952, *Heat Transfer Phenomena*, Wiley, New York, pp. 102-104.

4. Clausing, A.M., 1982, “Advantages of a Cryogenic Environment for Experimental Investigations of Convective Heat Transfer,” *International Journal of Heat and Mass Transfer*, Vol.25, pp.1255-1257.
5. Clausing, A.M., 1983, “Natural Convection Correlations for Vertical Surfaces Including Influences of Variable Properties,” ASME Journal of Heat Transfer, Vol.105, pp.138-143.
6. Fishenden, M., and Saunders, O.A., 1950, *An Introduction* to *Heat Transfer,* Oxford University Press, London, pp.95-99.
7. Fujii, T., and Imura, H., 1972, “Natural Convection Heat Transfer From a Plate With Arbitrary Inclination,” *International Journal of Heat and Mass Transfer,* Vol.15, pp.755-767.
8. Hassan, K.E., and Mohamed, S.A., 1970, “Natural Convection From Isothermal Flat Surfaces,” *International Journal Heat and* Mass *Transfer*, Vol. 13, pp. 1873-1886.
9. Lloyd, J.R., and Moran, W.R., 1974, “Natural Convection Adjacent to Horizontal Surfaces of Various Planforms,” ASME Journal of Heat Transfer, Vol. 96, pp.443-447.
10. Pera, L., and Gebhart, B., 1973, “Natural Convection Boundary Layer Flow Over Horizontal and Slightly Inclined Surfaces,” *International Journal of Heat and Mass Transfer*, Vol.16, pp. 1131-1146.
11. Rotem, L., and Claassen, L., 1969, “Natural Convection Above Unconfined Horizontal Surfaces,” *Journal of Fluid Mechanics,* Vol. 39, Part 1, pp. 173-192.

Nomenclature

A	=	area, m^2
c_p	=	specific heat, J/kg-K
f	=	defined by equation (4)
g	=	acceleration of gravity, m/s^2, or defined by equation (4)
h	=	heat transfer coefficient, W/m-K
L	=	characteristic length, m
m	=	mass, kg
t	=	time, s
T	=	temperature, K
T^*	=	$(T-T_\infty)/(T_o-T_\infty)$
V	=	free-stream velocity, m/s
α	=	thermal diffusivity, m^2/s
β	=	volume coefficient of expansion, K^{-1}
ε	=	emissivity
μ	=	dynamic viscosity, kg/m-s
ν	=	kinematic viscosity, m^2/s
ρ	=	density, kg/m^3
σ	=	Stefan-Boltzmann constant, W/m^2-K^4
Gr	=	Grashof number = $g\beta\Delta TL^3/\nu^2$
Nu	=	Nusselt number = hL/k

Published in the *ASME Journal of Heat Transfer*, November, 1989.

Pr	=	Prandtl number = ν/α
Ra	=	Rayleigh number = GrPr
Re	=	Reynolds number = VL/ν
Ri	=	Richardson number = $\mathrm{Gr}/\mathrm{Re}^2$

Subscripts

f	=	based on film temperature
o	=	initial condition
r	=	reference temperature
w	=	wall condition
∞	=	tunnel ambient condition

Superscripts

*	=	dimensionless quantity

Published in the *ASME Journal of Heat Transfer*, November, 1989.